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AUTOMATION OF PRODUCTION PROCESSES

Mathematical Modeling of the Hydrodynamics of the Bubble Mode during the Bottom Blowing of the Ladle Furnace: Report III

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Abstract—The formation and motion of gas bubbles in the melt substantially affect the heat exchange and kinetics of chemical transformations when performing the fire refining of copper in the ladle furnace. The variation in the bubble velocity, as well as of the volume and surface of the moving gas bubble over the melt height, is considered in the presented mathematical model.

Keywords: bubble, ladle furnace, copper refining, pore, blowing

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It was shown in [1] that, due to the nonuniformity in the distribution of the melt temperature and hydrostatic pressure over the bath of the ladle furnace, the volume of the floating bubble increases by a factor of 2.64. The variation in the volume and the surface of the gas bubble predetermined the problem on the development of the mathematical model of the bubble formation and its motion in the melt. These factors noticeably affect the heat exchange and kinetics of chemical transformations at the phase interface and in the liquid volume. Therefore, their evaluation, particularly as a function of the metal layer height, has applied value for determining the degree of using the gaseous reagent in the bubble as it floats and, correspondingly, the search for the rational level of a liquid bath in the furnace.

MOTION OF SINGLE BUBBLES

In the simplest case, the motion velocity can be established if we assume its stationarity. This is possibility determined by two similarity criteria, namely, the Weber number, which specifies the limiting possibilities of the conservation of the spherical shape of the bubble: $\mathrm{We} = \sigma/[g(\rho_l - \rho_g)D_b^2]$, and the Reynolds number: $\mathrm{Re} = \rho_g W D_b/\mu_l$. If this condition is fulfilled, the resistance force to the bubble motion and the lifting force should be equilibrated; i.e., the equality should be fulfilled:

$$\varsigma W^2/(\rho_l F) = V_b g(\rho_l - \rho_g). \quad (1)$$

Here, ς is the pressure resistance coefficient; F is the midsection area of the bubble, m^2; D_b and V_b are the bubble diameter, m, and volume, m^3; W is the bubble velocity, m/s; ρ_l is the liquid density, kg/m^3; ρ_g is the gas density in the bubble, kg/m^3; σ is the surface tension, N/m^2; and μ_l is the dynamic viscosity of liquid, MPa s.

From here,

$$W = [2g(\rho_l - \rho_g)V_b/(\varsigma F\rho_l)]^{1/2}. \quad (2)$$

Under the assumption that the bubble retains its spherical form, the velocity of its flotation is

$$W = [4g(\rho_l - \rho_g)D_b/(3\varsigma\rho_l)]^{1/2}. \quad (3)$$

Since $\rho_l \gg \rho_g$, expression (3) can be simplified to the relationship

$$W = [4gD_b/(3\varsigma)]^{1/2}. \quad (4)$$

Thus, the motion velocity of the bubble is largely determined by two factors in this case, namely, by the bubble size (D_b) and the coefficient of pressure resistance (ς), which in turn depends on the cross section. The smaller the bubble volume is, the closer its shape is to spherical. Due to the mobility of the phase interface, the gas bubble floats with a higher velocity than the solid particle or the drop of the same size, other conditions being the same. This is caused by the fact that liquid "adheres" to the solid surface and, consequently, it is immobile relative to it. On the contrary, the relative motion of the phases occurs at the liquid–gas phase interface. Therefore, smaller gradients of velocity are observed upon floating the bubble near its surface compared with the motion of the solid particle in similar conditions. In other words, the viscous friction more strongly affects the solid particle than the gas bubble. Rybchinskii and Hadamard theoretically derived a formula for determining the floating velocity of a spherical particle of any composition at Re < 2 [3]:

$$W = \frac{g(\rho_l - \rho_g)D_b^2}{6\mu_l}\frac{1 + \mu_g/\mu_l}{2 + 3\mu_g/\mu_l}. \quad (5)$$

For conditions of the copper melt $\mu_g \ll \mu_l$; therefore,

$$W = \frac{g(\rho_l - \rho_g)D_b^2}{12\mu_l}. \quad (6)$$

Let us find the approximate velocity of bubble floating. For this purpose, we take into account the fact that the dynamic viscosity of the copper melt depends on temperature according to the expression found in [4]:

$$\log\mu_l = 800T^{-1} - 0.05 \ \ [\text{MPa s}]. \quad (7)$$

In this case, at an average melt temperature of 1473 K [1], we correspondingly find $\mu_l = 3.112$ MPa s and $\rho_l = 8215$ kg/m^3. We accept that the average bubble diameter is (10 + 17.15)/2 = 13.57 mm [1]. In this case,

$$W = 9.81(8215 - 1.27)(0.014)^2/(12 \times 3, 11 \times 10^6)$$
$$= 0.39 \times 10^{-6} \text{ m/s}.$$

Consequently,

$$\text{Re} = \rho_g W D_b/\mu_g = 1.27 \times 0.39 \times 10^{-6}$$
$$\times 0.014/(52.3 \times 10^{-6}) = 0.131 \times 10^{-3},$$
$$\text{We} = \sigma/[g(\rho_l - \rho_g)D_b^2]$$
$$= 1.35 \times 9.81/[9.81 \times (8215 - 1.27)(0.14)^2] = 0.892.$$

Thus, in the case of the stationary process of the formation and floatation of the bubble, the size of the latter cannot increase above 17.15 mm, which contradicts the practical observations. It seems likely that gas saturating the melt is intensely suctioned to the bubble and no stationary floating occurs. On the other hand, the found similarity criteria indicate that the bubble remains spherical.

As the bubble size increases, the influence of surface tension forces decreases compared with the dynamic effect of the melt, and the shape deviates from spherical approaching the hydrodynamically unstable oblate spheroid. The motion velocity of such bubbles can be calculated starting from the energy conservation law. The work of decreasing the thickness of the oblate spheroid by magnitude dh equals the variation in energy caused by the surface tension. This variation in energy equals to the surface tension (σ) multiplied to the velocity increment (dF). Thus,

$$\varsigma W^2/2(\rho_l F dh) = -\sigma dF. \quad (8)$$

The minus sign in Eq. (8) is caused by the fact that an increase in cross-section F is accompanied by a decrease in the bubble height. The bubble volume does not vary during its deformation; i.e., $V_b = Fh$ = const. Therefore,

$$dV_b = Fdh + hdF = 0. \quad (9)$$

Substituting Fdh by $-hdF$ in expression (8), we derive

$$h = V_b/F = 2\sigma/(\varsigma\rho_l W^2). \quad (10)$$

Substituting found value V_b/F into (2), we have

$$W = [4g\sigma(\rho_l - \rho_g)/(\varsigma^2\rho_l^2)]^{1/2}. \quad (11)$$

This formula was derived by the author of [5]. The Re dependence of the resistance coefficient for the bubble flowing around by liquid can be found in various publications. Particularly, for Re > 50, good results are give by the Moore formula [6]:

$$\varsigma = \frac{48}{\text{Re}(1 - 2.21/\sqrt{\text{Re}})}. \quad (12)$$

At Re > 100, magnitude ς changes comparatively weakly. Since the Reynolds number increases as D_b increases, the floating velocity of large bubbles should depend weakly on the size. This is confirmed by the calculation of V_b of air bubbles in water, depending on D_b. With average bubble size $\bar{D}_b = 13.57$ mm, its floating velocity is almost 2 mm/s; i.e., the bubble "shoots" the ladle furnace less than for 0.75 s.

It is known [7] that, if the bubble volume exceeds 2 cm^3, it becomes shaped like almost a regular spherical segment and floats in a liquid with any viscosity with the velocity

$$W = (1 \pm 0.05)\sqrt{dG_b/2}. \quad (13)$$

Accepting $\bar{D}_b = 0.01357$ m, we find according to formula (13) $W = 0.26$–0.27 m/s, which substantially differs from the previously found data.

A considerable divergence of the results indicates the low probability of the occurrence of the stationary floating mode of the bubble, which implies an analysis of the regularities of its nonstationary motion in the melt.

REGULARITIES OF THE NONSTATIONARY BUBBLE MOTION

The nonstationary character of the motion of the gas bubble is caused by the external forces affecting it and the variation in geometric sizes. With the mathematical description of the bubble motion through the melt layer in the ladle furnace with blowing "from below," the following assumptions were accepted:

(i) the inertia forces, which affect the bubble from the side of the gas flow during blowing through the orifice in the bottom of the ladle furnace are negligibly small;

(ii) the geometric shape of the bubble during its motion through the melt layer is spherical and is retained to the instant of its contact with the melt surface;

(iii) the instantaneous hydrodynamic resistance, which occurs in the case of the bubble motion with large acceleration, can be neglected because of the essential melt viscosity;

(iv) we assume at this stage of the investigation that the melt is immobile;

(v) the inertial component of the resistance force, which is caused by the attached bubble mass, is almost absent.

The bubble moves in the melt volume and precisely the melt resists its motion. Therefore, the resistance coefficient should be determined by the melt properties, and since it is expressed through the Reynolds number, it should also include (in addition to the motion velocity and the characteristic size) the physical properties of the melt. When taking into account the proper melt motion, the Reynolds number should be calculated by the relative motion velocity of the bubble. Therefore, the motion equation for a single bubble, allowing for all factors and our assumptions, can be written as follows:

$$m_b dW/d\tau = V_b g(\rho_l - \rho_g) - fF_{sur}\rho_l W^2/2 - (\varsigma\pi D_b^2/4)(\rho_l W^2/2), \tag{14}$$

where m_b is the bubble weight, kg; F_{sur} is its surface area, m^2; and f is the viscosity friction coefficient.

After simple algebraic transformations and accepting $\varsigma = 0.47$ [8] and $d\tau = dh/W$, we derive

$$(\rho_g/\rho_l)W(dW/dh) = g(1-\rho_g/\rho_l) - (3W^2/D_b)(f+0.47/4). \tag{15}$$

Denoting $A = \rho_g/\rho_l$, $B = g(1 - \rho_g/\rho_l)$, and $C = 3(f + 0.47/4)D_b$, we have the differential equation of motion of a single bubble of the following form:

$$WdW/dh = B/A - W^2C/A.$$

Let $B/A = k(h)$, $C/A = m(h)$; then we derive the equation in a form of the differential Bernoulli equation

$$WdW/dh + m(h)W^2 = k(h). \tag{16}$$

Equation (16) is solved by the substitution $W = uz$. In the issue, we have the analytical solution to the equation of motion of the bubble in the form

$$W = \exp\left\{-\int_0^h \dot{m}(h)dh\sqrt{2\left(\int_0^h k(h)\exp\left[2\int_0^h m(h)dh\right]dh + C\right)}\right\}, \tag{17}$$

where C is the integration constant, which is found from condition $h = 0$, $W = 0$.

For the numerical evaluation of the motion velocity of the bubble in the melt volume over the height of the ladle furnace, let us find coefficients A, B, and C.

1. $A = \rho_g/\rho_l$.

For the copper melt [4] in range $T = 1373–1873$ K, we have

$$\rho_l = 8300 - 0.73(T - 1356) \text{ [kg/m}^3\text{]}.$$

Using the previously found data on the temperature field in the ladle furnace [1], we derive $\rho_l = 9090.59 - 1024.19\exp(-0.181h)$,

$$\rho_g = m_g/V_b = \rho_{g0}V_0/V_b = \rho_{g0}(V_0/V_b).$$

If we take into account that the axis of projections coincides with the direction of the vector of the motion velocity of the bubble, then, after substituting $x = 1.5 - h$ into expression V_b/V_0 [1], we finally derive

$$\rho_g = \frac{1.27}{1 - 1.76x - 6.89\ln[0.64 + 0.24(1.5-x)] + [2.70/1 + 0.38(1.5-x) - 1.71]}. \tag{18}$$

2. $B = 9.81(1 - A)$.

3. $C = 3(f + 0.47/4)D_b(h)$.

Coefficient f can be calculated according to the Al'tshul' formula [7]:

$$f = 0.074/\mathrm{Re}^{0.2}.$$

If we accept Re < 400, which corresponds to all possible motion modes of the bubble, then f is of about 0.07–0.02.

Then the numerical value of $(f + 0.47/4)$ lies in limits of 0.14–0.19. Let us take this quantity equal 0.16.

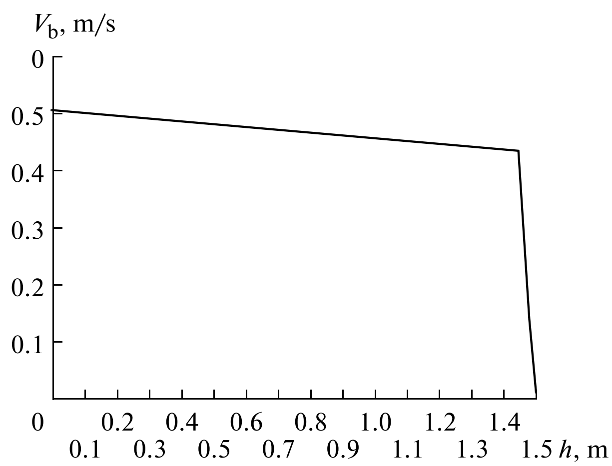
Further, we have

$$D_b = (6V_b/3.14)^{1/3} = (1.19V_b)^{1/3} = (1.91 \times 423 \times 10^{-9}V_b/V_0)^{1/3}, \tag{19}$$

where 423×10^{-9} is the initial volume of the bubble, m^3.

Using equation [1]

$$V_b/V_0 = 1 - 1.76x - 6.89\ln[0.64 + 0.24(1.5-x)] + [2.69/(1 + 0.38(1.5-x)) - 1.71],$$



Distribution of the floating velocity of the bubble over the melt depth.

we finally derive

$$k(h) = B/A$$
$$= g[(\rho_l - \rho_g)/\rho_l]/(\rho_l/\rho_g) = g(\rho_l/\rho_g - 1),$$
$$m(h) = C/A = 3 \times 0.16/(D_b\rho_g/\rho_l)$$
$$= 0.48\rho_l/(D_b\rho_g).$$

It is seen from the preliminary analysis of solution (17) of the differential equation of motion of a single bubble that its velocity has a clearly pronounced nonlinear character over the melt height in the ladle furnace. It seems to be impossible to formalize the integrals entering solution (17) through the known functions; therefore, let us find the approximated solution by the method of pseudoconstant coefficients. Results of calculations are shown in the figure.

The analysis of the data of the figure indicates that the shape of the plot in the gas-input region depends on the computational step. Particularly, the coordinates of the inflection point correspond to points (1.45, 0.44) and (1.45, 0.19). As the computational step increases, with the conservation of the ordinates of the inflection points at the previous level, abscissas shift to 1.5. In other words, the change in the bubble velocity from 0 to 438 m/s occurs almost instantly (in the limits of 10^{-3} m). In the main region of the ladle, the floating velocity of the bubble increases almost according to the quadratic parabola law.

CONCLUSIONS

The use of the developed mathematical model allowed us to show the formation and motion character of the gas bubble over the height of the melt layer. It is established that the maximal bubble acceleration during its floating is observed in the detachment instant from the "nozzle" (80% of the limiting velocity at the output from metal). Its maximal motion velocity at the outcrop on the melt surface is 0.507 m/s by the results of calculations. Thus, starting from the instant of the motion onset to the leg break, the bubble passes a distance of 10^{-3}–2×10^{-3} m, and its motion velocity changes from zero to 0.44 m/s for this time. In this range, its shape is close to the ellipsoid of revolution, the large axis of which coincides with the direction of its velocity vector. Further, the shape of the gas bubble becomes close to spherical and its velocity, varying according to the quadratic parabola law, increases approximately to 0.51 m/s.

The data found on the motion rate of the bubble in the initial region of gas input into the melt are confirmed by the results of modern investigations of hydro-gas-dynamics incorporation processes of the gas into the liquid. The experimental data indicate that the bubble is connected with the orifice through which the gas enters the melt by the gas bridge ("leg"). According to [9], the bubble detaches at the instant the critical volume of gases by the force of the leg breaking is attained. The leg stays in the detachment instant and is the nucleus of a new bubble. Such a mechanism of formation of a gas phase is also confirmed by the results of investigations found previously [10] in experiments with the use of rapid filming, which was taken into account to adopt the selected model.

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